Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

9140549920

FURTHER MATHEMATICS **9231/21**

Paper 2 Further Pure Mathematics 2 **May/June 2024**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

DC (SL) 340584

1 Find the roots of the equation $z^3 = -108\sqrt{3} + 108\mathrm{i}$, giving your answers in the form $r\left(\cos\theta + \mathrm{i}\sin\theta\right)$, where $r > 0$ and $0 < \theta < 2\pi$. [5]

2 Find the Maclaurin's series for $e^{1+x^2} + e^{1-x}$ up to and including the term in x^2. [4]

3 It is given that

$$x = \sin^{-1} t \quad \text{and} \quad y = t\cos^{-1} t, \quad \text{for } 0 \leqslant t < 1.$$

(a) Show that $\dfrac{dy}{dx} = -t + \sqrt{1-t^2}\cos^{-1} t$. [3]

(b) Find $\frac{d^2y}{dx^2}$ in terms of t. [4]

4 It is given that, for $n \geqslant 0$, $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \, dx$.

(a) Show that, for $n \geqslant 2$,

$$(n-1)I_n = \left(\frac{3}{5}\right)^{n-2}\left(\frac{4}{5}\right) + (n-2)I_{n-2}.$$ [5]

[You may use the result that $\frac{d}{dx}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$.]

(b) Find the value of I_4. [3]

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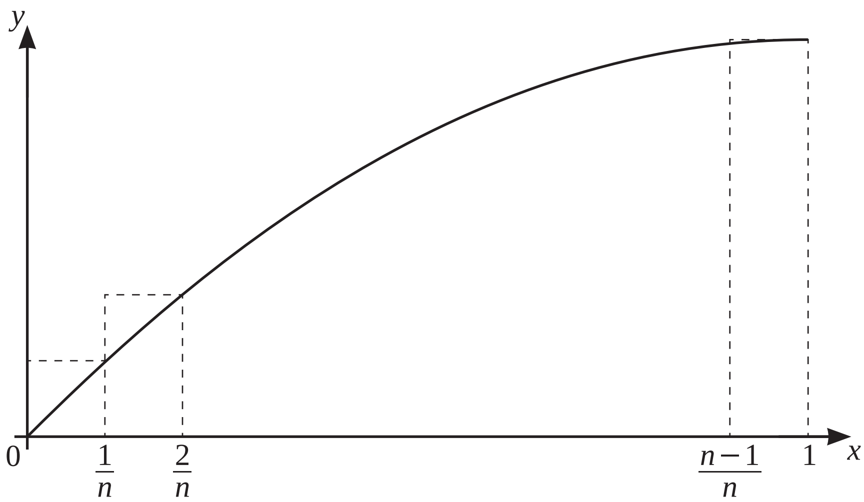

The diagram shows the curve with equation $y = 2x - x^2$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 (2x - x^2)\,dx < U_n$, where

$$U_n = \left(1 + \frac{1}{n}\right)\left(\frac{2}{3} - \frac{1}{6n}\right).$$ [5]

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(b) Use a similar method to find, in terms of n, a lower bound L_n for $\int_0^1 (2x - x^2)\,dx$. [4]

(c) Show that $\lim_{n \to \infty} (U_n - L_n) = 0$. [2]

6 **(a)** Show that $(\cosh x + \sinh x)^{\frac{1}{2}} = e^{\frac{1}{2}x}$. [2]

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(b) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5(\cosh x + \sinh x)^{\frac{1}{2}},$$

given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$. [10]

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7 **(a)** Use the substitution $u = 1 + x^2$ to find

$$\int \frac{x}{\sqrt{1+x^2}}\,\mathrm{d}x.$$ [2]

(b) Find the solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x^2 \sinh^{-1} x,$$

given that $y = 1$ when $x = 1$. Give your answer in the form $y = \mathrm{f}(x)$. [10]

8 **(a)** Find the set of values of a for which the system of equations

$$\begin{aligned} 6x + ay \qquad &= 3, \\ 2x - \ y \qquad &= 1, \\ x + 5y + 4z &= 2 \end{aligned}$$

has a unique solution. [2]

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(b) Show that the system of equations in part **(a)** is consistent for all values of a. [3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 6 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 5 & 4 \end{pmatrix}.$$

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $(14\mathbf{A}+24\mathbf{I})^2 = \mathbf{PDP}^{-1}$. [7]

(d) Use the characteristic equation of $\mathbf{A}$ to show that

$$(14\mathbf{A}+24\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A}+b\mathbf{I})^2 ,$$

where b is an integer to be determined. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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