Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

914054992

FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 20 pages. Any blank pages are indicated.

where $r > 0$ and $0 < \theta < 2\pi$.	$=-108\sqrt{3}+108i$, giving your answers in the fo	[:

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3	It is	given	that
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	$x = \sin^{-1} t$ and $y = t \cos^{-1} t$, for $0 \le$	<i>t</i> < 1.
(a)	Show that $\frac{dy}{dx} = -t + \sqrt{1 - t^2} \cos^{-1} t$.	[3]

Find $\frac{d^2y}{dx^2}$ in terms of t					
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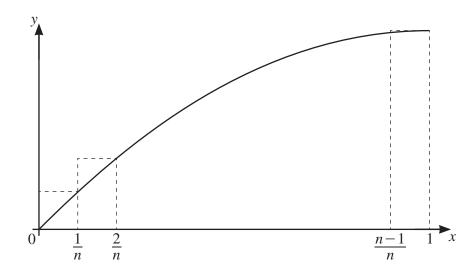
- 4 It is given that, for $n \ge 0$, $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \, dx$.
 - (a) Show that, for $n \ge 2$,

	$^{-2}\left(\frac{4}{5}\right) + (n-2)I_{n-2}.$	[5]
[You may use the result that $\frac{d}{dx}(\operatorname{sech} x) =$	$-\tanh x \operatorname{sech} x$.]	

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(b)	Find the value of I_4 .	[3]

5



The diagram shows the curve with equation $y = 2x - x^2$ for $0 \le x \le 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a)	By considering the sum of the areas of these rectangles, show t	that $\int_0^1 (2x-x^2) dx < U_n$, where	
	$U_{n} = \left(1 + \frac{1}{n}\right)\left(\frac{2}{3} - \frac{1}{6n}\right).$	[:	5

(b)	Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 (2x - x^2) dx$. [4]
()	
(c)	Show that $\lim_{n \to \infty} (U_n - L_n) = 0$. [2]

6	(a)	Show that $\left(\cosh x + \sinh x\right)^{\frac{1}{2}} = e^{\frac{1}{2}x}$.	[2]
	(b)	Find the particular solution of the differential equation	
		$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$	
		$dx^2 = dx$	
		given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$.	[10]
			[10]
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7	(a)	Use the substitution $u = 1 + x^2$ to find	
		$\int \frac{x}{\sqrt{1+x^2}} \mathrm{d}x.$	[2]
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	(b)	Find the solution of the differential equation $x\frac{dy}{dx} - y = x^2 \sinh^{-1}x,$	
		$x\frac{d}{dx} - y = x^2 \sinh^{-x} x,$	
		given that $y = 1$ when $x = 1$. Give your answer in the form $y = f(x)$.	[10]
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8	(a)	Find the set of values of a for which the system	of equations
		6x + ay	= 3,
		2x-y	= 1,
		x+5y+4z	= 2
		has a unique solution.	[2
	(b)	Show that the system of equations in part (a) is of	consistent for all values of a . [3]

The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 6 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 5 & 4 \end{pmatrix}.$$

Find a matrix P and a diagonal matrix D such that $(14\mathbf{A} + 24\mathbf{I})^2 = \mathbf{PDP}^{-1}$.	[7]
	•••••••

[4]

(d)	Use the characteristic equation of A to show that	
	$(14\mathbf{A} + 24\mathbf{I})^2 = \mathbf{A}^4 (\mathbf{A} + b\mathbf{I})^2,$	
	where b is an integer to be determined.	

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.		

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