



# Cambridge International AS & A Level

CANDIDATE  
NAME

--

CENTRE  
NUMBER

--	--	--	--	--

CANDIDATE  
NUMBER

--	--	--	--



## FURTHER MATHEMATICS

**9231/21**

Paper 2 Further Pure Mathematics 2

**May/June 2024**

**2 hours**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

- 1** Find the roots of the equation  $z^3 = -108\sqrt{3} + 108i$ , giving your answers in the form  $r(\cos \theta + i \sin \theta)$ , where  $r > 0$  and  $0 < \theta < 2\pi$ . [5]

[illegible]

**2** Find the Maclaurin's series for  $e^{1+x^2} + e^{1-x}$  up to and including the term in  $x^2$ . [4]

[illegible]

**3** It is given that

$$x = \sin^{-1}t \quad \text{and} \quad y = t \cos^{-1}t, \quad \text{for } 0 \leq t < 1.$$

(a) Show that  $\frac{dy}{dx} = -t + \sqrt{1-t^2} \cos^{-1} t$ . [3]

[illegible]

(b) Find  $\frac{d^2y}{dx^2}$  in terms of  $t$ .

[4]

[illegible]

**4** It is given that, for  $n \geq 0$ ,  $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \, dx$ .

**(a)** Show that, for  $n \geq 2$ ,

$$(n-1)I_n = \left(\frac{3}{5}\right)^{n-2} \left(\frac{4}{5}\right) + (n-2)I_{n-2}. \quad [5]$$

[You may use the result that  $\frac{d}{dx}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$  .]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. There are no margins, text, or other markings on the page.

(b) Find the value of  $I_4$ .

[3]

[illegible]





(b) Use a similar method to find, in terms of  $n$ , a lower bound  $L_n$  for  $\int_0^1 (2x - x^2) dx$ . [4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(c) Show that  $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$ . [2]

[illegible]

- 6 (a) Show that  $(\cosh x + \sinh x)^{\frac{1}{2}} = e^{\frac{1}{2}x}$ . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5(\cosh x + \sinh x)^{\frac{1}{2}},$$

given that, when  $x = 0$ ,  $y = 1$  and  $\frac{dy}{dx} = \frac{4}{3}$ . [10]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 7 (a) Use the substitution  $u = 1 + x^2$  to find

$$\int \frac{x}{\sqrt{1+x^2}} dx. \quad [2]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \sinh^{-1} x,$$

given that  $y = 1$  when  $x = 1$ . Give your answer in the form  $y = f(x)$ . [10]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

- 8 (a) Find the set of values of  $a$  for which the system of equations

$$6x + ay = 3,$$

$$2x - y = 1,$$

$$x + 5y + 4z = 2$$

has a unique solution.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Show that the system of equations in part (a) is consistent for all values of  $a$ .

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



(d) Use the characteristic equation of  $\mathbf{A}$  to show that

$$(14\mathbf{A} + 24\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A} + b\mathbf{I})^2,$$

where  $b$  is an integer to be determined.

[4]

[illegible]



## Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]





**BLANK PAGE**

---

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at [www.cambridgeinternational.org](http://www.cambridgeinternational.org) after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.